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Gravitational Perturbation Induced by an Intense Laser Pulse

Peiyong Ji,[1] Shi-tong Zhu,[2] and Wen-da Shen[1]

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The energy-level shifts of the hydrogen spectrum in curved spacetime induced by intense short laser pulses are studied. With present high-power laser pulses the magnitude of the energy-level shifts of highly excited hydrogen atom should be detectable.

1. INTRODUCTION

General relativity and some other metric theories predict that gravitation will cause curved spacetime. Dealing with large-scale spacetime, however, there are few experiments available for gravitation compared to other interactions. With the development of techniques for high-power lasers, the intensity of laser pulses can reach the magnitude of the atomic unit ($>3 \times 10^{16}$ W/cm^2) and even 10^{20} W/cm^2. In addition, present detecting techniques can provide micrometer space resolution and picosecond time resolution. Therefore it is feasible to detect the gravitational effect of an intense laser pulse and find related detecting methods. The gravitational coupling between laser beams was examined by Scully (1979) via the Einstein–Maxwell equations, and the amplitude and phase variations of a probe pulse due to a high-intensity laser pulse were obtained. It was shown by Parker (1980, 1982) that the energy levels of the atomic spectrum in curved spacetime would be shifted due to the local space-time curvature. In this paper we study the energy-level shifts for the hydrogen atom in curved space produced by intense laser pulses and seek a way to check the validity of the theory of general relativity.

[1] Department of Physics, Shanghai University, Shanghai, 201800, China; e-mail: pyji@public.sta.net.cn.
[2] Shanghai Institute of Optics and Fine Mechanics, Academia Sinica, Shanghai, 201800, China.

2. METRIC INDUCED BY AN INTENSE LASER FIELD

One takes the electromagnetic fields for a laser pulse to have the following forms (Scully, 1979):

$$E_2(\mathbf{r}, t) = \mathscr{E}(\mathbf{r}, t)\sin(\omega t - kx) \tag{2.1a}$$

$$B_3(\mathbf{r}, t) = \frac{v}{c^2}\mathscr{E}(\mathbf{r}, t)\sin(\omega t - kx) \tag{2.1b}$$

$$B_1(\mathbf{r}, t) = \left[1 - \left(\frac{v}{c}\right)^2\right]^{1/2} \frac{\mathscr{E}(\mathbf{r}, t)}{c}\cos(\omega t - kx) \tag{2.1c}$$

where $\mathscr{E}(\mathbf{r}, t)$ is the envelope of the pulse, with

$$\mathscr{E}^2(\mathbf{r}, t) = E_0^2 A[\theta(v(t + T_0) - x) - \theta(vt - x)]\delta(y)\delta(z) \tag{2.2}$$

E_0 is the amplitude of the pulse, A is the section of the beam, v is the velocity of the pulse, T_0 is the duration of the pulse, and the step function is

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

The gravitational metric can be obtained by solving the Einstein equations

$$g_{\mu\nu}(\mathbf{r}, t) = \eta_{\mu\nu} + h_{\mu\nu}(\mathbf{r}, t) \tag{2.3}$$

where

$$\eta_{\mu\nu} = \begin{Bmatrix} c^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Bmatrix} \tag{2.4}$$

and

$$h_{\mu\nu}(\mathbf{r}, t) = h(\mathbf{r}, t)M_{\mu\nu} \tag{2.5}$$

with

$$h(\mathbf{r}, t) = -\frac{2G\varepsilon_0 E_0^2 A}{c^2} \times \ln\left\{\frac{v(t + T_0) - x + [(v(t + T_0) - x)^2 + (1 - v^2/c^2)(y^2 + z^2)]^{1/2}}{vt - x + [(vt - x)^2 + (1 - v^2/c^2)(y^2 + z^2)]^{1/2}}\right\} \tag{2.6}$$

and

$$M_{\mu\nu} = \begin{pmatrix} 1 & -v/c^2 & 0 & 0 \\ -v/c^2 & v^2/c^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1/c^2)[1 - (v/c)^2] \end{pmatrix} \tag{2.7}$$

In the short-pulse approximation, (2.6) reduces to

$$h(\mathbf{r}, t) = \frac{-(2\varepsilon_0 G E_0^2 A v T_0 / c^2)}{[(x - vt)^2 + (1 - v^2/c^2)(y^2 + z^2)]^{1/2}} \tag{2.6a}$$

Restricted by the uncertainty principle, the section of the laser beam is limited to the range

$$A_0 \leq y^2 + z^2 \leq A$$

where A_0 denotes the minimum section of the laser beam allowed by the uncertainty principle, and A is the effective section of the beam. Limiting to geodesic motion along the x direction and substituting into equation (2.6a) with $A_1 = (y^2 + z^2)$, one has

$$h(x, t) = \frac{-(2\varepsilon_0 G E_0^2 A v T_0 / c^2)}{[(x - vt)^2 + (1 - v^2/c^2)A_1]^{1/2}} \tag{2.6b}$$

The line element characterizing the spacetime of the short laser is

$$ds^2 = \left(1 + \frac{h}{c^2}\right)c^2\, dt^2 - 2\frac{v}{c^3} h(c\, dt)\, dx + \left(-1 + \frac{v^2}{c^4} h\right) dx^2 - dy^2 + \left[-1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right] dz^2 \tag{2.8}$$

Setting $\bar{x} = x - vt$, $\bar{t} = t - F(\bar{x})$, $\bar{y} = y$, $\bar{z} = z$, and selecting $F(\bar{x})$ to satisfy the identity

$$\frac{\partial F}{\partial \bar{x}} = \frac{v + (v/c^2)h - (v^3/c^4)h}{c^2 - v^2 + h - 2(v^2/c^2)h + (v^4/c^4)h} \tag{2.9}$$

we find that the coupled term $(c\, dt)\, dx$ disappears and then (2.8) reads

$$ds^2 = \left(1 - \frac{v^2}{c^2}\right)\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right](c\, dt)^2 - \left(1 - \frac{v^2}{c^2}\right)^{-1} (d\bar{x})^2 - (d\bar{y})^2 - \left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right](d\bar{z})^2 \tag{2.10}$$

In new coordinates ($c\bar{t}$, $\bar{x}$, $\bar{y}$, $\bar{z}$), therefore, the metric components have the following forms:

$$\bar{g}_{00} = \left(1 - \frac{v^2}{c^2}\right)\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right] \tag{2.11}$$

$$\bar{g}_{11} = -\left(1 - \frac{v^2}{c^2}\right)^{-1} \tag{2.12}$$

$$\bar{g}_{22} = -1 \tag{2.13}$$

$$\bar{g}_{33} = -\left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right] \tag{2.14}$$

where

$$h = \frac{-2\varepsilon_0 G E_0^2 A v T_0/c^2}{[\bar{x}^2 + (1 - v^2/c^2)A_1]^{1/2}} \tag{2.6c}$$

3. GEODESIC MOTION AND FERMI NORMAL COORDINATES

Taking account of the energy-level shifts of the hydrogen atom in Fermi normal coordinates (Parker, 1980, 1982), we will examine the geodesic motion determined by the metric (2.10) and then set up the corresponding Fermi normal coordinates. The Lagrangian matching the metric (2.10) is

$$L = \left(1 - \frac{v^2}{c^2}\right)\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right](c\dot{\bar{t}})^2 - \left(1 - \frac{v^2}{c^2}\right)^{-1}(\dot{\bar{x}})^2 - (\dot{\bar{y}})^2 - \left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right](\dot{\bar{z}})^2 \tag{3.1}$$

and it satisfies

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{\bar{x}}^\mu}\right) - \frac{\partial L}{\partial \bar{x}^\mu} = 0 \qquad (\mu = 0, 1, 2, 3) \tag{3.2}$$

Here $\bar{x}^\mu = d\bar{x}^\mu/(cd\tau)$, $ds = c\, d\tau$, and τ is the proper time. Solving the Lagrangian equations, one gets

$$c\bar{t} = \frac{C_0}{(1 - v^2/c^2)[1 + (1 - v^2/c^2)h/c^2]} \tag{3.3}$$

$$\bar{y} = C_y \tag{3.4}$$

$$\bar{z} = \frac{C_z}{1 - (1 - v^2/c^2)h/c^2} \tag{3.5}$$

and

$$\bar{x} = \pm\left(1 - \frac{v^2}{c^2}\right)^{1/2} \left\{\frac{C_0^2}{(1 - v^2/c^2)[1 + (1 - v^2/c^2)\, h/c^2]} - 1 - (C_y)^2 - \frac{C_z^2}{1 - (1 - v^2/c^2)h/c^2}\right\}^{1/2} \tag{3.6}$$

where C_0, C_y, and C_z are integration constants. The solutions indicate that the atom acted upon by the gravitational field of the intense laser will move along a geodesic with velocity $\bar{x}$, $\bar{y}$, and $\bar{z}$.

Taking the velocity along the direction of x, and choosing y and z to be zero when $\bar{x} \to \pm\infty$ ($h \to 0$), one can get the integrals $C_y = C_z = 0$, and $C_0^2 = (1 - v^2/c^2)$. Therefore equations (3.3) and (3.6) read

$$c\bar{t} = \frac{\pm 1}{(1 - v^2/c^2)^{1/2}[1 + (1 - v^2/c^2)h/c^2]} \tag{3.3a}$$

and

$$\bar{x} = \pm\left(1 - \frac{v^2}{c^2}\right)^{1/2} \left\{\frac{-(1 - v^2/c^2)h/c^2}{1 + (1 - v^2/c^2)h/c^2}\right\}^{1/2} \tag{3.6a}$$

From equations (3.3a) and (3.6a), the velocity of the particle in the direction of $\bar{x}$ can be found as

$$V_x = \frac{d\bar{x}}{c\, d\bar{t}} = \left(1 - \frac{v^2}{c^2}\right)\left[-\left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{1/2}\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{1/2} \tag{3.7}$$

and the nonzero components of the affine connection read

$$\Gamma^0_{01} = \frac{-(1 - v^2/c^2)\bar{x}h}{2c^2[1 + (1 - v^2/c^2)h/c^2][\bar{x}^2 + (1 - v^2/c^2)A_1]} \tag{3.8}$$

$$\Gamma^1_{00} = -\frac{(1 - v^2/c^2)^3\bar{x}h}{2c^2[\bar{x}^2 + (1 - v^2/c^2)A_1]} \tag{3.9}$$

$$\Gamma^1_{33} = -\frac{[1 - v^2/c^2)^2\bar{x}h}{2c^2[\bar{x}^2 + (1 - v^2/c^2)A_1]} \tag{3.10}$$

and

$$\Gamma^3_{31} = \frac{(1 - v^2/c^2)\bar{x}h}{2c^2[\bar{x}^2 + (1 - v^2/c^2)A_1][1 - (1 - v^2/c^2)h/c^2]} \tag{3.11}$$

The nonzero components of the Riemann curvature tensors are written as

$$R_{1010} = R_{0101} = -R_{1001} = -R_{0110}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]} \times \left\{4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)hc^2 + \left(3\bar{x}^2 - \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)h^2 - 2\left(1 - \frac{v^2}{c^2}\right)^4 A_1h^2\right\} \tag{3.12}$$

$$R_{3030} = R_{0303} = -R_{3003} = -R_{0330}$$

$$= -\frac{\left(1 - \frac{v^2}{c^2}\right)^4\bar{x}^2h^2}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2} \tag{3.13}$$

$$R_{1313} = R_{3131} = -R_{3113} = -R_{1331}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)\frac{h}{c^2}\right]} \times\left\{4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)hc^2 - \left(3\bar{x}^2+\frac{2A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)h^2 + 2\left(1-\frac{v^2}{c^2}\right)^3 A_1h^2\right\} \tag{3.14}$$

$$R_{00} = R^1_{010} + R^3_{030}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]} \times\left\{-4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^3 hc^2 + 2\left(\bar{x}^2+\frac{A_1c^2}{h}\right)\left(1-\frac{v^2}{c^2}\right)h^2 + 4\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^5\frac{h^3}{c^2} - 2\left(1-\frac{v^2}{c^2}\right)^6 A_1\frac{h^3}{c^2}\right\} \tag{3.15}$$

$$R_{11} = R^0_{101} + R^3_{131}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1-\frac{v^2}{c^2}\right)A_1\right]^2\left[1-\left(1-\frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2} \times\left\{-10\bar{x}^2\left(1-\frac{v^2}{c^2}\right)h^2 + 4\left(1-\frac{v^2}{c^2}\right)^3 A_1h^2 + 6\bar{x}^2\left(1-\frac{v^2}{c^2}\right)^4\frac{h^4}{c^4} - 4\left(1-\frac{v^2}{c^2}\right)^5 A_1\frac{h^4}{c^4}\right\} \tag{3.16}$$

$$R_{33} = R^{0}_{303} + R^{1}_{313}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]} \times \left\{-4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^2 hc^2 + 2\left(\frac{A_1c^2}{h} - \bar{x}^2\right)\left(1 - \frac{v^2}{c^2}\right)^3 h^2 + 4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^4\frac{h^3}{c^2} - 2\left(1 - \frac{v^2}{c^2}\right)^5 A_1\frac{h^3}{c^2}\right\} \tag{3.17}$$

$$R = R^0_0 + R^1_1 + R^3_3$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2} \times \left\{22\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^3 h^2 - 8\left(1 - \frac{v^2}{c^2}\right)^4 A_1h^2 - 14\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^5\frac{h^4}{c^4} + 8\left(1 - \frac{v^2}{c^2}\right)^6 A_1\frac{h^4}{c^4}\right\} \tag{3.18}$$

Along with the geodesic in the direction of $\bar{x}$, a Fermi normal basis can be selected as (Manasse and Misner, 1963)

$$\hat{e_0} = c\bar{t}\,\frac{\partial}{c\partial t} + \bar{x}\,\frac{\partial}{\partial\bar{x}} \tag{3.19}$$

$$\hat{e_1} = \alpha\,\frac{\partial}{c\partial\bar{t}} + \beta\,\frac{\partial}{\partial\bar{x}} \tag{3.20}$$

$$\hat{e_2} = \frac{\partial}{\partial y} \tag{3.21}$$

$$\hat{e_3} = \left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{-1/2}\frac{\partial}{\partial\bar{z}} \tag{3.22}$$

and, obviously,

$$e_0^{\bar{t}} = c\bar{t}, \qquad e_0^{\bar{x}} = \bar{x}, \qquad e_1^{\bar{t}} = \alpha, \qquad e_1^{\bar{x}} = \beta, \qquad e_2^{\bar{y}} = 1$$
$$e_3^{\bar{z}} = \left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^{-1/2} \tag{3.23}$$

where

$$\alpha^2 = \frac{-h}{c^2[1 + (1 - v^2/c^2)h/c^2]^2} \tag{3.24}$$

and

$$\beta^2 = \frac{1 - v^2/c^2}{1 + (1 - v^2/c^2)h/c^2} \tag{3.25}$$

Riemann curvature tensors in the Fermi normal basis can be correspondingly written as

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}} = R_{\hat{1}\hat{0}\hat{1}\hat{0}} = -R_{\hat{0}\hat{1}\hat{1}\hat{0}} = -R_{\hat{1}\hat{0}\hat{0}\hat{1}}$$
$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v_2}{c^2}\right)A_1\right]^2\left[1 + \left(1 - \frac{v^2}{c^2}\right)\frac{h}{c^2}\right]^2} \tag{3.26}$$
$$\times \left\{4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^2 hc^2 + \left(3\bar{x}^2 - \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^3 h^2 - 2\left(1 - \frac{v^2}{c^2}\right)^4 A_1h^2\right\}$$

$$R_{\hat{0}\hat{3}\hat{0}\hat{3}} = R_{\hat{3}\hat{0}\hat{3}\hat{0}} = -R_{\hat{0}\hat{3}\hat{3}\hat{0}} = -R_{\hat{3}\hat{0}\hat{0}\hat{3}}$$
$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2\frac{h^2}{c^4}\right]^2} \tag{3.27}$$
$$\times \left\{-5\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)h^2 + 2\left(1 - \frac{v^2}{c^2}\right)A_1h^2 + 3\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^5\frac{h^4}{c^3} - 2\left(1 - \frac{v^2}{c^2}\right)^6 A_1\frac{h^4}{c^4}\right\}$$

$$R_{\hat{1}\hat{3}\hat{1}\hat{3}} = R_{\hat{3}\hat{1}\hat{3}\hat{1}} = -R_{\hat{1}\hat{3}\hat{3}\hat{1}} = -R_{\hat{3}\hat{1}\hat{1}\hat{3}}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2 \frac{h^2}{c^4}\right]^2} \times \left\{4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^2 hc^2 + \left(\bar{x}^2 - \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^3 h^2 - 2\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^4 \frac{h^3}{c^2} + \left(-\bar{x}^2 + \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^5 \frac{h^4}{c^4}\right\} \qquad (3.28)$$

$$R_{\hat{0}\hat{0}} = R^{\hat{1}}_{\hat{0}\hat{1}\hat{0}} + R^{\hat{3}}_{\hat{0}\hat{3}\hat{0}} = -R_{\hat{1}\hat{0}\hat{1}\hat{0}} - R_{\hat{3}\hat{0}\hat{3}\hat{0}}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[- \left(1 - \frac{v^2}{c^2}\right)^2 \frac{h^2}{c^4}\right]^2} \times \left\{-4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^2 hc^2 + \left(10\bar{x}^2 + \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^3 h^2 + \left(2\bar{x}^2 - \frac{4A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^4 \frac{h^3}{c^2} + \left(-6\bar{x}^2 - \frac{2A_1c^2}{h}\right) \times \left(1 - \frac{v^2}{c^2}\right)^5 \frac{h^4}{c^4} + 4\left(1 - \frac{v^2}{c^2}\right)^6 A_1 \frac{h^4}{c^4}\right\} \qquad (3.29)$$

$$R_{\hat{1}\hat{1}} = R^{\hat{0}}_{\hat{1}\hat{0}\hat{1}} + R^{\hat{3}}_{\hat{1}\hat{3}\hat{1}} = R_{\hat{0}\hat{1}\hat{0}\hat{1}} - R_{\hat{1}\hat{3}\hat{1}\hat{3}}$$

$$= \frac{1}{4c^4\left[\bar{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2 \frac{h^2}{c^4}\right]^2} \times \left\{-6\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^3 h^2 + 2\left(1 - \frac{v^2}{c^2}\right)^4 A_1 h^2 + 4\bar{x}^2\left(1 - \frac{v^2}{c^2}\right)^5 \frac{h^4}{c^4} - 2\left(1 - \frac{v^2}{c^2}\right)^6 A_1 \frac{h^4}{c^4}\right\} \qquad (3.30)$$

$$R_{\hat{3}\hat{3}} = R^{\hat{0}}_{\hat{3}\hat{0}\hat{3}} + R^{\hat{1}}_{\hat{3}\hat{1}\hat{3}} = R_{\hat{0}\hat{3}\hat{0}\hat{3}} - R_{\hat{1}\hat{3}\hat{1}\hat{3}}$$

$$= \frac{1}{4c^4\left[\overline{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)^2 \frac{h^2}{c^4}\right]^2}$$
$$\times\left\{ - 4\overline{x}^2\left(1 - \frac{v^2}{c^2}\right)^2 hc^2 + \left(-6\overline{x}^2 + \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^3 h^2\right.$$
$$+\left(2\overline{x}^2 + \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^4 \frac{h^3}{c^2}$$
$$\left. + \left(4\overline{x}^2 - \frac{2A_1c^2}{h}\right)\left(1 - \frac{v^2}{c^2}\right)^5 \frac{h^4}{c^4} - 2\left(1 - \frac{v^2}{c^2}\right)^{s6} A_1\frac{h^4}{c^4}\right\} \qquad (3.31)$$

$$R = R^{\hat{0}}_{\hat{0}} + R^{\hat{1}}_{\hat{1}} + R^{\hat{3}}_{\hat{3}} = R_{\hat{0}\hat{0}} - R_{\hat{1}\hat{1}} - R_{\hat{3}\hat{3}}$$

$$= \frac{1}{4c^4\left[\overline{x}^2 + \left(1 - \frac{v^2}{c^2}\right)A_1\right]^2\left[1 - \left(1 - \frac{v^2}{c^2}\right)\frac{h^2}{c^4}\right]^2}\left\{22\overline{x}^2\left(1 - \frac{v^2}{c^2}\right)^3 h^2\right. \qquad (3.32)$$
$$\left.-8\left(1 - \frac{v^2}{c^2}\right)^4 A_1h^2 - 14\overline{x}^2\left(1 - \frac{v^2}{c^2}\right)^5\frac{h^4}{c^4} + 8\left(1 - \frac{v^2}{c^2}\right)^6 A_1 \frac{h^4}{c^4}\right\}$$

It is obvious that the Riemann curvature scalar in the Fermi normal basis (3.32) is the same as the identity (3.18).

4. GRAVITATIONAL SHIFTS OF THE ENERGY LEVELS OF THE HYDROGEN ATOM

Now one can evaluate the gravitational shifts, using the expression obtained in the previous section for the Riemann curvature tensors in normal coordinates and the results for hydrogen energy-level shifts (Parker, 1980, 1982). It follows from (2.6b) that $h(x, t)$, with units of the square of velocity, gets its maximum value at $\overline{x}^2 = 0$, and one can expect that the energy-level shifts for the hydrogen atom located at that position would be observable. Considering $h/c^2 << 1$, one can retain the lowest order for $h(\mathbf{r}, t)$ in equations (3.26)–(3.32) and then the expressions are rewritten, in $\overline{x}^2 = 0$, as

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}} = R_{\hat{1}\hat{0}\hat{1}\hat{0}} = -R_{\hat{0}\hat{1}\hat{1}\hat{0}} = -R_{\hat{1}\hat{0}\hat{0}\hat{1}} \simeq -\frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right)h \tag{4.1}$$

$$R_{\hat{0}\hat{3}\hat{0}\hat{3}} = R_{\hat{3}\hat{0}\hat{3}\hat{0}} = -R_{\hat{0}\hat{3}\hat{3}\hat{0}} = -\ R_{\hat{3}\hat{0}\hat{0}\hat{3}} \simeq 0 \tag{4.2}$$

$$R_{\hat{1}\hat{3}\hat{1}\hat{3}} = R_{\hat{3}\hat{1}\hat{3}\hat{1}} = -R_{\hat{1}\hat{3}\hat{3}\hat{1}} = -R_{\hat{3}\hat{1}\hat{1}\hat{3}} \simeq -\frac{1}{2C^2 A_1}\left(1 - \frac{v^2}{c^2}\right)h \tag{4.3}$$

$$R_{\hat{0}\hat{0}} \simeq \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right)h \tag{4.4}$$

$$R_{\hat{1}\hat{1}} \simeq 0 \tag{4.5}$$

$$R_{\hat{3}\hat{3}} \simeq \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right)h \tag{4.6}$$

$$R \simeq 0 \tag{4.7}$$

Substituting the above curvature tensors into the expression for the energy-level shifts (Parker, 1980, 1992)

$$E^{(1)} = \mathscr{A} R_{\hat{0}\hat{0}} + \mathscr{B} R + \sum_{i=1}^{3} \mathscr{C}^{ii} R_{\hat{0}\hat{i}\hat{0}\hat{i}}$$

we obtain

$$E^{(1)} = \mathscr{A} R_{\hat{0}\hat{0}} + \mathscr{C}^{11} R_{\hat{0}\hat{1}\hat{0}\hat{1}} = \frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right)h[\mathscr{A} - \mathscr{C}^{11}] \tag{4.8}$$

where $\mathscr{A}$ and $\mathscr{C}^{11}$ are constants that depend on the fine structure constant and the energy of the electron. For atoms in either highly excited states or Rydberg states, the energy levels have the forms

$$E_n^{(1)} \simeq 2.5 \frac{\hbar^2}{\alpha^2 m_e}\left[\frac{1}{2c^2 A_1}\left(1 - \frac{v^2}{c^2}\right)h\right]n^4 \tag{4.9}$$

where α is the fine structure constant, and

$$E_n^{(0)} = -2\pi\hbar c R_y n^{-2} \tag{4.10}$$

where R_y is the Rydberg constant. The ratio of the gravitational shifts to the differences between successive energy levels is

$$\frac{E_{n+1}^{(1)} - E_n^{(1)}}{E_{n+1}^{(0)} - E_n^{(0)}} \simeq 5.26 \times 10^{-16} \frac{G\varepsilon_0 E_0^2 A v T_0}{c^4 A_1^{3/2}} \left(1 - \frac{v^2}{c^2}\right)^{1/2} n^6 \qquad (4.11)$$

For present high-power laser techniques involving megajoule energies on a picosecond time scale, one can choose the following parameters:

$$\frac{1}{2}\varepsilon_0 E_0^2 A v T_0 = 2.5 \times 10^6 \text{ J}, \qquad v = 0.9c, \qquad n = 10^2$$

and by considering the restriction due to the uncertainty principle, for a laser with $\lambda \sim 10^{-7}$ m, take $A_1 = 10^{-12}$ m^2. Then we have

$$\frac{\Delta E_n^{(1)}}{\Delta E_n^{(0)}} \simeq 10^{-24} \qquad (4.12)$$

From the literature (Scully, 1979) it is feasible to reach the above sensitivity by means of present laboratory techniques.

5. SUMMARY AND CONCLUSIONS

We have investigated the gravitational perturbation for the hydrogen spectrum and estimated the energy output and the traveling velocity of high-power laser pulses required for observable gravitational effects, and therefore may have found a probe to check whether Einstein's gravitational theory is valid within laboratory dimensions.

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